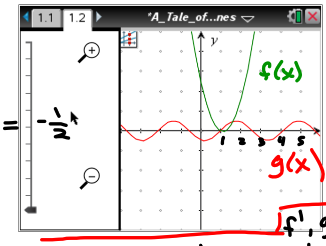
8.2 L'Hopital's Rule

$\frac{y_1}{y_2}$ $\frac{m_1}{m_2} = \frac{f'}{g'}$

$\lim_{x\to 1} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} = -\frac{1}{2}$



$f(1)=0$

$g(1)=0$

$\frac{0}{0}$?

Indeterminate

$\boxed{f', g' \text{ exist; if } f(a)=g(a)=0,\ g'(a)\neq 0 \quad \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}}$

Jan 6-9:43 PM

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

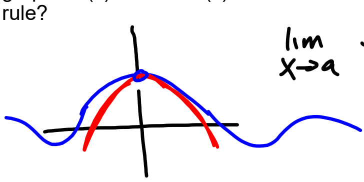
$\frac{0}{0}$

Jan 3-9:21 AM

$$\lim_{x\to 0}\frac{\sin(x)}{x} = \frac{\cos(0)}{1} = 1$$

$\frac{0}{0}$

graph sin(x)/x and cos(x)/1. How does this support l'hopital's rule?



$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

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$$\lim_{x\to 0}\frac{\sqrt{1+x}}{x} = \cancel{\lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}}{1} = \frac{1}{2}}$$

∞

$f(a)=g(a)=0$

$\cancel{1 \neq 0} = 0$

$$\lim_{x\to 0^+}\frac{1}{x} = \infty$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1-x/2}{x^2} =$$

$\frac{0}{0}$

$$\lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - 0 - \frac{1}{2}}{2x} = \lim_{x\to 0}\frac{-\frac{1}{4}(1+x)^{-3/2}}{2} = -\frac{1}{8}$$

$\frac{0}{0}$

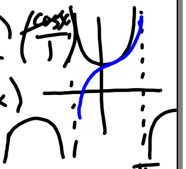
Jan 6-9:47 PM

$\frac{\infty}{\infty}$ if $\lim_{x \to a} \frac{f(x)}{g(x)}$ is $\frac{\infty}{\infty}$ indet. use l'hopital's rule

$$\lim_{x \to \pi/2} \frac{\sec x}{1+\tan x} = \lim_{x \to \frac{\pi}{2}} \frac{\cancel{\sec x} \tan x}{\sec^{\cancel{2}} x} \qquad \left(\frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{\cos x}}}\right)$$

$\frac{\infty}{\infty}$

$$= \lim_{x \to \frac{\pi}{2}} \sin x = \boxed{1}$$



$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}} = \lim_{x \to \infty} \frac{\frac{1}{x}}{2 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}} = \lim_{x \to \infty} \frac{\sqrt{x}}{x} \qquad \frac{x^{\frac{1}{2}}}{x^{1}}$$

$\frac{\infty}{\infty}$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{x}} = 0$$

$$\lim_{x \to \infty} x \sin \frac{1}{x} = \lim_{x \to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$$

$\infty \cdot 0$

$\frac{0}{0}$

$$= \lim_{x \to \infty} \frac{\cancel{-\frac{1}{x^2}} \cdot \cos \frac{1}{x}}{\cancel{-\frac{1}{x^2}}} = \cos 0$$

$$= 1$$

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = \lim_{x \to 1} \frac{(x-1) - \ln x}{\ln x\,(x-1)}$$

$\infty - \infty$ $\qquad \frac{0}{0}$

$$\lim_{x \to 1} \frac{\left(1 - 0 - \frac{1}{x}\right) x}{\left(\ln x \cdot 1 + (x-1)\frac{1}{x}\right) x} = \lim_{x \to 1} \frac{x-1}{x \ln x + (x-1)}$$

$\frac{0}{0}$

$$\lim_{x \to 1} \frac{1}{x \cdot \frac{1}{x} + \ln x + 1} = \frac{1}{2}$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = y \qquad (e)$$

$$\lim_{x \to \infty} \ln\left(1 + \frac{1}{x}\right)^x = \ln y$$

$$\lim_{x \to \infty} x \ln\left(1 + \frac{1}{x}\right) = \ln y$$

$\frac{0}{0}$

$$\lim_{x \to \infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = \ln y$$

$$\lim_{x \to \infty} \frac{\frac{1}{\left(1 + \frac{1}{x}\right)} \cdot \left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}}$$

$$e^{1} = e^{\ln y}$$

$$e = y$$